Computational Finance, Fall 2016
Computer Lab 5

The aim of the Lab is to learn to learn to determine market parameters from the prices of traded options.

If we make assumptions about the market behavior or about the methods of option pricing, we get functions that for a given set of market (or option pricing) parameters give theoretical values of every concrete option. Black-Scholes formulas are examples of such functions that for given value of the volatility σ and for given put or call option parameters (exercise price, duration) give the price of the option. Whenever we have such functions (which can be explicit formulas or some computer programs that compute the prices) and there are available prices of some traded options we can try to determine the unknown parameters from the known option prices. More precisely, suppose that we know the current prices $V_1, V_2, \ldots, V_m$ of m different options and that $f_i(\theta)$ are the functions that give the option prices for the (unknown) market parameters θ. Then we have m equations:

$$f_i(\theta) = V_i, \; i = 1, \ldots, m.$$

Usually the number of unknown market parameters is much smaller than the number of available option prices, so the system of equations may be solved in the least squares sense by minimizing the function

$$F(\theta) = \frac{1}{2}\sum_{i=1}^{m}(f_i(\theta) - V_i)^2.$$

Let us use the current information about ask prices of 5.5-months call options (expiration on March 17,2017) for Apple stock available from from `http://finance.google.com`.

Exercise 1. (Implied volatility) Let us assume that the Black-Scholes market model with constant volatility holds, then call option prices can be computed by Black-Scholes formula. Assume $r = 0.02$ and $D = 0$, then the only unknown parameter is σ. Since we have only one unknown parameter, only one equation is needed to determine the value of σ and if the assumption about the market model is correct, then every known option price should give the same value of σ. Use the equation solver `fsolve` from `scipy.optimize` to find a value of σ for each of 10 actual option prices corresponding to 10 strike prices closest to the current share price. Show the dependence of found σ values on the exercise price on a graph. In order to do this, for each value of the exercise price define a function of one argument σ that computes the difference of the corresponding theoretical call option price and the observed price of the option and use this function as an input for the command `optimize.fsolve` together with a suitable initial guess for the parameter σ.

Exercise 2. Define a function that for a given value of σ computes the sum of squares of differences of the theoretical and observed option prices and use a minimizer from `scipy.optimize` to find the least squares estimate of σ. For this σ, find the largest difference between the theoretical and observed option prices.

Exercise 3. Consider Black-Scholes market model with the non-constant volatility

$$\sigma(s,t) = |\theta_0 + \theta_1 \arctan(0.02 \cdot (s - 102))|.$$

From the course web page you can download a module `lab5solver` that contains a function `lab5solver(theta,E,S0)` that for a given values of E and $S0$ and for a given parameter vector θ computes the theoretical price of a call option. Use the function and the option price data to find suitable values of θ_0 and θ_1.

Homework 2 (deadline Oct. 5, 2016) Consider the dividend adjusted closing prices of Bank of America Corporation stock for the period 27.09.2015-26.09.2016 (obtainable from `http://finance.yahoo.com/`, historical prices, column with the label `Adj Close`).

1. Consider Black-Scholes model with non-constant trend

$$\mu(t) = \theta_0 + \theta_2 \log(\frac{S(t)}{S(t-2\Delta t)}),$$

where Δt is the length of one trading day in years, and non-constant volatility

$$\sigma(s,t) = |\theta_1 + \frac{\theta_3}{1+0.1s}|$$

Find maximal likelihood estimates for the parameters $\theta_0, \theta_1, \theta_2$ and θ_3 by using the approach of discussed in the lecture (that is based on using the Euler's approximation of the equation for $dS(t)$ directly). Can we assume that the observed stock prices follow the market model with the parameters μ and σ given by the formulas above? Explain!

2. Use the the average of bid and ask prices from the file BAC_call.txt (available in Moodle) as the observed call option prices to compute the least squares estimate of the volatility by assuming that a BS model with constant volatility holds. Compute corresponding theoretical option prices for all strike prices which appear in the file. Plot the difference of the theoretical and observed option prices on a graph. NB! Your code should use the file of option prices as it is in the Moodle (without modifying it manually)!

All comments and explanations should be included (together with copies of obtained numerical results) as Python comments in your solution file.