Lab4_sol

October 3, 2017

1 Computational Finance, Lab 4, 2017

1.1 Exercise 1

Download from http://finance.yahoo.com/ historical prices of the stock of Apple corporation for one year as a *.csv file. Write a script, that reads the adjusted closing prices from the file and uses them to estimate μ and σ (assuming the BS model with constant parameters holds). NB! pay attention to the order the data is in the file!

Solution. Let us read the data in. Note that the adjusted closing prices are in the sixth column and that there is a header row. To get the same results, the data should be for the period from September 26, 2016 to September 25, 2017. This corresponds to a full year data.

```
In [3]: import numpy as np
        prices=np.loadtxt("h:/compfin_labs/AAPL.csv",delimiter=",",usecols=(5,),skiprows=1)
        n=np.size(prices)
        print(n)
        print(prices[:5])# first 5 observations from the file
        print(prices[-5:])# last five observations
```

```
252
[ 110.923363  111.129723  111.974823  110.235504  111.090431]
[ 158.729996  156.070007  153.389999  151.889999  150.550003]
```

Note that the data in the file is in natural order (oldest first). So we can use it without reordering.

In stock price modelling time is usually measured in years, which is divided into working days (holidays and weekends are usually ignored). We have a full year of data with 252 observations, so the length of the time interval between working days is $\frac{1}{252}$

```
In [6]: delta_t=1/n
        S=prices #a shorter name for data
        i=np.arange(0,n-1)
        x=np.log(S[i+1]/S[i]) #or log(S[1:]/S[:-1])
        sigma=np.std(x)/np.sqrt(delta_t)
        mu=np.mean(x)/delta_t+sigma**2/2
        print("mu =",mu,", sigma=",sigma)
```

```
mu = 0.321177081128 , sigma= 0.170319644136
```

1.2 Exercise 2

Read the help information of the normality tests `stats.anderson` and `stats.shapiro`. Check if we can assume that the Apple stock prices follow the Black-Scholes model with constant coefficients.

Solution From the lecture we know that if the stock price follows BS market model with constant trend and volatility, then the values of the vector x computed in the previous exercise should correspond to iid values from a Normal distribution. Each normality test computes a value of a specially constructed measure (statistic) for which it is known what are reasonable values in the case of data from Normal distribution.

```
In [7]: from scipy import stats
        print(stats.shapiro(x))

(0.9281308054924011, 1.0944208872487593e-09)
```

The probability to get a value of the statistic as large (or larger) as we got in the case of normally distributed data is very low So it is very unlikely that the market data corresponds to the BS model with constant coefficients

```
In [8]: print(stats.anderson(x))

AndersonResult(statistic=3.5989025275522124, critical_values=array([ 0.567,  0.646,  0.775,  0
```

The probability to get values larger than 1.075 is 0.01. The value of statistic is much larger, so this test also tells us that it is very unlikely that the market data corresponds to BS model with constant coefficients.

1.3 Exercise 3

Assume that

$$\mu_\theta(t_i) = \theta_0 + \theta_1 \cdot \frac{S_i - S_{i-2}}{S_i}$$

and

$$\sigma_\theta(s,t) = \theta_2.$$

Use the maximal likelihood method to determine the optimal values of θ.

Solution We have to take into account that the first observation for which we can compute μ corresponds to $i = 3$ and that the last time moment for which we can compute Y_i (see the lab handout) is $i = n - 2$. We use this information to define the function $f(\theta)$ (the negative of the likelihood function).

```
In [9]: def f(theta):
            #assume that S and delta_t are defined outside
            i=np.arange(2,n-1)
            Y= np.log(S[i+1]/S[i])
            mu=theta[0]+theta[1]*(S[i]-S[i-2])/S[i]
            sigma_values=theta[2]
            m=(mu-sigma_values**2/2)*delta_t
            return sum( (Y-m)**2/(2*sigma_values**2*delta_t)+np.log(sigma_values) )
```

Note that if we take $\theta_0 = \mu$, $\theta_1 = 0$, $\theta_2 = \sigma$ in the case of constant μ and σ, we get a market model with constant coefficients. This gives a way to determine a set of reasonable starting values for optimization

```
In [12]: from scipy import optimize
         theta_opt1=optimize.fmin(f,[0.32,0,0.17])
         print(theta_opt1)
```

```
Optimization terminated successfully.
         Current function value: -315.430485
         Iterations: 33
         Function evaluations: 64
[  3.14220004e-01   1.98709642e-04   1.70881377e-01]
```

Note that different starting values may give different results. The best one is with the lowest value of the function we are trying to minimize

```
In [13]: theta_opt2=optimize.fmin(f,[0.0,1,0.3])
         print(theta_opt2)
```

```
Optimization terminated successfully.
         Current function value: -313.808880
         Iterations: 38
         Function evaluations: 75
[ 0.00133007  1.11049117  0.17189769]
```

The value of the function we are minimizing is large, so from the two results, the first one is a better result

```
In [14]: optimize.fmin(f,[0.1,-0.2,0.4])
```

```
Optimization terminated successfully.
         Current function value: -315.430095
         Iterations: 91
         Function evaluations: 165
```

```
Out[14]: array([ 0.3146524 , -0.16763341,  0.17088049])
```

Again a different result, but not better than the first one.

1.4 Exercise 4

We can again test the validity of our assumptions. Namely, if our assumptions are correct, then $\frac{Y_i - m_i}{\sigma_\theta(S_i, t_i)}$ should be independent random variables from a normal distribution for the estimated parameters θ. So we can again use the tests for normality. Please test the validity of the model fitted in the previous exercise.

Solution