Computational Finance, Fall 2017
Computer Lab 12

Suppose we want to find the current prices of an European option so that the error at the current stock price $S(0) = S_0$ was less that ε. When using a finite difference method for the untransformed equation, we have to fix only one artificial boundary; a good starting point is to take $s_{max} = 2 \cdot S_0$ (if σ is large or the time period is long, it may make sense to take larger value for s_{max}). In the case of solving the transformed equation, we also have to introduce another boundary for which we may take $s_{min} = \frac{S_0}{2}$ and define $x_{min} = \ln s_{min},\ x_{max} = \ln s_{max}$. Our procedure is as follows:

1. Fix a starting value n_0 and choose m_0 (in the case of the explicit method choose it from the stability constraint) and define $z = 1$.
2. Solve the problem with a finite difference method (starting with $n = z \cdot n_0$ and $m = m_0$) and estimate the error by Runge's method, until the (estimated) finite difference discretization error is less than $\frac{\varepsilon}{2}$.
3. multiply s_{max} by 2, divide s_{min} by 2, increase z by one in the case of the transformed equation or multiply it by two in the case of the untransformed equation and solve the problem with the same method (starting with $n = z \cdot n_0,\ m = m_0$ again until the (estimated) finite difference discretization error is less than $\frac{\varepsilon}{2}$. If the answer changes by more than $\frac{\varepsilon}{2}$ then repeat the step. Otherwise we assume that we have obtained the solution with the desired accuracy.

Note that the variable z is chosen so that for each new value of ρ, the step size Δx is the same for the first computation. This is important for the error estimate to work correctly.

Exercise 1. Use the procedure above to compute the price of the call option with accuracy 0.02 by using the explicit finite difference method for the transformed problem and simple boundary conditions in the case $S_0 = 51,\ E = 50,\ r = 0.03,\ D = 0,\ T = 1,\ \sigma = 0.5$. Find the actual error of the final answer.

In the lecture we derived Crank-Nicolson method for solving option pricing problems, where at each time step we have to solve the system of equations

$$\begin{aligned} U_{0k} &= \phi_1(t_k), \\ a_{ik}U_{i-1,k} + b_{ik}U_{ik} + c_{ik}U_{i+1,k} &= d_{ik}U_{i-1,k+1} + e_{ik}U_{i,k+1} + f_{ik}U_{i+1,k+1},\ i = 1, \dots, n-1, \\ U_{nk} &= \phi_2(t_k), \end{aligned}$$

where

$$\begin{aligned} a_{ik} &= -\frac{\alpha(x_i, t_k + \frac{\Delta t}{2})\Delta t}{2\Delta x^2} + \frac{\beta(x_i, t_k + \frac{\Delta t}{2})\Delta t}{4\Delta x}, \\ b_{ik} &= 1 + \frac{\alpha(x_i, t_k + \frac{\Delta t}{2})\Delta t}{\Delta x^2} + \frac{r\Delta t}{2}, \\ c_{ik} &= -\frac{\alpha(x_i, t_k + \frac{\Delta t}{2})\Delta t}{2\Delta x^2} - \frac{\beta(x_i, t_k + \frac{\Delta t}{2})\Delta t}{4\Delta x}, \\ d_{ik} &= -a_{ik},\ f_{ik} = -c_{ik}, \\ e_{ik} &= 1 - \frac{\alpha(x_i, t_k + \frac{\Delta t}{2})\Delta t}{\Delta x^2} - \frac{r\Delta t}{2}. \end{aligned}$$

When using the method for untransformed equation then x denotes s (corresponds to stock price, not to the logarithm of the stock price) and

$$\alpha(x,t) = \frac{(\sigma(x,t)x)^2}{2}, \ \beta(x,t) = (r - D)x.$$

Additionally, the final condition is used as follows: $U_{i,m} = p(x_i)$.

Exercise 2. Find the price of the European option with payoff

$$p(s) = \begin{cases} 40 - \frac{s}{2}, & s \leq 80, \\ 0, & 80 < s \leq 110, \\ \frac{3s}{2} - 165, & s > 110 \end{cases}$$

with maximal error 0.01 for current stock price $S0 = 105$ in the case, $r = 0.05$, $D = 0$, $T = 0.5$ and nonconstant volatility

$$\sigma(s,t) = 0.4 + \frac{0.3}{1 + 0.01(s - 90)^2}.$$

Use Crank-Nicolson method for the untransformed equation, the exact boundary condition

$$\phi_1(t) = p(0)e^{-r(T-t)}$$

at the boundary $x_{min} = 0$ and the boundary condition corresponding to a special (linear in s) solution at the boundary $x = s_{max}$ to obtain the answer.