Computational Finance, Fall 2017
Computer Lab 13

The aim of the lab is to learn to compute prices of American options.

If we have reduced the Black-Scholes option pricing equation to a problem of the form

$$\frac{\partial u}{\partial t}(x,t)+\alpha(x,t)\frac{\partial^2 u}{\partial x^2}(x,t)+\beta(x,t)\frac{\partial u}{\partial x}(x,t)-r\,u(x,t),\ x\in(x_{min},x_{max}),0\le t<T,$$
$$u(x_{min},t)=\phi_1(t),0\le t<T,$$
$$u(x_{max},t)=\phi_2(t),0\le t<T,$$
$$u(x,T)=u_0(x),\ x\in(x_{min},x_{max}).$$

and have derived a finite difference method for computing option prices, then approximate prices of the corresponding American option can be computed by taking maximum of found approximate prices U_{ik} and the values of $u_0(x_i)$ at each time step. The convergence rate of the resulting method is $O(\Delta t+\Delta x^2)$ even in the case of Crank-Nicolson method. The transformed equation is of this form with $u_0(x)=p(e^x)$, $\alpha(x,t)=\frac{\sigma(e^x,t)^2}{2}$ and $\beta(x,t)=r-D-\alpha(x,t)$; the untransformed equation (if we rename s to x) is also of this form with $u_0(x)=p(x)$, $\alpha(x,t)=\frac{x^2\sigma(x,t)^2}{2}$, $\beta(x,t)=(r-D)\cdot x$.

Exercise 1. Modify solvers using the basic implicit methods for computing American options by solving original and transformed BS equation. Use the methods for computing approximate prices of the American put option in the case $r=0.1$, $\sigma=0.5$, $D=0$, $T=0.5$, $E=100$, $S0=100$, $p(s)=\max(E-s,0)$ and for $n=10,20,40,80,160$, $m=5,20,80,320,1280$ (5 computations in total). Use x_{max} that corresponds to the maximal stock price $2*S0$ and in the case of solving the transformed problem, $x_{min}=\ln\frac{S0}{2}$. In each case, compute the logarithms of the absoculte values of differences between consecutive results and present them on a graph. Can we say that the values lie approximately on a straight line? If we can, what is the slope of the line?

Exercise 2. Find the price of the American put option considered in the previous exercise with maximal error 0.01 for current stock price $S0=100$.

Exercise 3. If we use the transformed equation for finding approximate prices, we can approximate the derivative of the option price with respect to s by

$$\frac{\partial v}{\partial s}(S(0),0)=\frac{1}{S(0)}\frac{\partial u}{\partial x}(\ln S(0),0)\approx\frac{1}{S(0)}\frac{U_{i+1,0}-U_{i-1,0}}{2\Delta x},$$

where i is such that $x_i=\ln S(0)$. If we use the untransformed equation for finding approximate prices, we can approximate the derivative of the option price with respect to s by

$$\frac{\partial v}{\partial s}(S(0),0)\approx\frac{U_{i+1,0}-U_{i-1,0}}{2\Delta s},$$

where i is such that $s_i=S(0)$. If we use our standard methods of choosing m and n values, then the discretization error of those approximations is $O(\Delta x)$ for the transformed equation and $O(\Delta s)$ for the untransformed equation and hence is reduced approximately by two when we multiply n by 2. Find the value $\frac{\partial v}{\partial s}(100,0)$ of the price v of American put option with maximal error 0.001 for the option considered in the previous exercise by using the basic implicit method for solving untransformed BS equation.

Homework 6. (December 5, 2017). Assume that the Black-Scholes market model with constant volatility $\sigma=0.5$ holds. Assume that $r=0.02$, $D=0$, $S(0)=45$. Use the solver for Crank-Nicolson method for transformed BS equation to find the value of the derivative of the option price with respect to stock price at time $t = 0$ when exercise time is $T = 0.5$ and the payoff function is given by

$$p(s) = \begin{cases} \frac{(s-40)^2}{10}, & \text{if } 40 \le s < 50 \\ 20 - \frac{(s-60)^2}{10}, & \text{if } 50 \le s < 70 \\ \frac{(s-80)^2}{10}, & \text{if } 70 \le s < 80 \\ 0 & \text{otherwise} \end{cases}$$

both in the case of European and American option. The derivatives should be computed with total error less than 0.01. Note that the convergence rate of CN method is different for European and American options, so use Runge's estimate correctly!