Simulation methods in mathematical finance
Sample final examination

Consider an option which gives the owner the the payoff $\max(S(T) - 1.2 \cdot S(T_1), 0)$ at $T = 0.5$. Assume that the risk free interest rate is 0.1 and that the current value of the stock is $S(0) = 20$. Assume also that the option price at time $t = 0$ is given by the expected value

$$E[e^{-rT} \max(S(T) - 1.2S(T_1), 0)],$$

where $S(t)$ follows a suitable market model specified later. Hint: Note that the Brownian motion values in the formulas for $S(T)$ and $S(T_1)$ are not independent!

1. Assume BS model with constant volatility $\sigma = 0.5$ holds and that $T_1 = 0.2$. Compute the price of the option with accuracy 0.01 with standard MC method. The answer should be approximately 1.065

2. Use control variates method with the discounted payoff of the standard call option a the control variate to compute the price of the option considered in the previous exercise with accuracy 0.001.

3. Assume that the stock price satisfies the system of stochastic differential equations

$$dS(t) = S(t) \cdot (r\,dt + V(t)dB_1(t)),$$
$$dV(t) = (0.5 - V(t))\,dt + 0.5 \cdot V(t)\,dB_2(t)$$

   with the initial conditions $S(0) = 20,\ V(0) = 0.3$. Here B_1 and B_2 are independent standard Brownian motions. Let $T_1 = 0.2$. Find the price of the option by the standard MC method with Monte Carlo error less than 0.01 with the probability 0.95 by using Euler's method with $m = 10$ time steps for generating stock prices. The answer should be 0.527 ± 0.01.

4. Modify the code of the third problem so that it works with any value of T_1 so that T_1 is one of time moments for which the stock price is computed. Derive a suitable vector v for stratified sampling for this option. Use optimal stratified sampling for computing the value of the option with m=77 and $T_1 = \frac{1}{\sqrt{7}}$ and allowed MC error 0.001.