Simulation Methods in Financial Mathematics
Computer Lab 14

Goal of the lab:

- To learn a way to compute derivatives of the option price with respect to market parameters

Often the behaviour of the random variable X depends on some parameter θ and therefore the expected value $E(g(X(\theta))$ depends also on that parameter, so we have

$$V(\theta) = E[g(X(\theta))]$$

and often one is interested in the value of the derivative of the expected value with respect to that parameter. For example, stock price (and hence the price of an option) depends on the initial stock price $S(0)$, the volatility σ, risk free interest rate r etc and we may want to know how the price changes if some of the parameters changes. A general way to compute the derivatives is to use finite difference approximations, for example

$$V'(\theta) = \frac{V(\theta+h) - V(\theta-h)}{2h} + O(h^2).$$

When using Monte-Carlo method for computing the values of V it is important not to compute $V(\theta+h)$ and $V(\theta-h)$ independently, but to write

$$\frac{V(\theta+h) - V(\theta-h)}{2h} = E[\frac{g(X(\theta+h)) - g(X(\theta-h))}{2h}]$$

and to compute this expected value by MC by using the same random variables for generating both the values of $X(\theta+h)$ and $X(\theta-h)$ at the same time.

Let us consider an European call option with $E = 100$, $T = 0.5$, assume $r = 0.05$, $D = 0$, $S(0) = 100$ and that the Black-Scholes market model holds.

Tasks:

1. Assume that the volatility is constant: $\sigma = 0.5$. Implement Monte-Carlo method for computing the derivative of the price of the option with respect to $S(0)$, using the exact formula for generating the stock prices. Compute the derivative at $S(0) = 100$ using $h = 20$ and allowed MC error 0.001 (corresponding to $\alpha = 0.05$).

2. If we want to compute the derivative with a given total error then we have to estimate the error coming from the choice of h. For this the Runge's method can be used. We assume that error coming from the choice of h is $C \cdot h^2$ with an unknown C. In order to estimate C we do two computations, one for $h = h_0$ and the other on with $h = \frac{h_0}{2}$ (where h_0 is some value we choose) and using the difference of the answers it is possible to estimate C. After that we can choose h so that $C \cdot h^2 \leq \frac{\epsilon}{2}$, where ϵ is the allowed total error and do the final computation with this value of h and allowed MC error $\frac{\epsilon}{2}$. Find the derivative of the option price considered in the previous problem with total error less than 0.0005 with probability 0.95.

3. **Homework 7** (deadline May 17, 2017) Consider the case of BS model with non-constant volatility $\sigma(s) = 0.5 + \frac{\sigma_1}{1+(s-100)^2 * 0.01}$, where $\sigma_1 = 0.3$. Implement a procedure for computing the value of the derivative of the option price with the payoff function

$$p(s) = \max\{20 - |s - 105|, 0\}$$

with respect to σ_1 with a given accuracy, assuming $r = 0.05$, $D = 0$, $S(0) = 100$, $T = 0.5$. Compute the answer with total error 0.2. Note that the there are now three sources of the total error: the error with respect to m, the error depending on h and the Monte-Carlo error.