Computational Finance

Lecture 2

September 10, 2018

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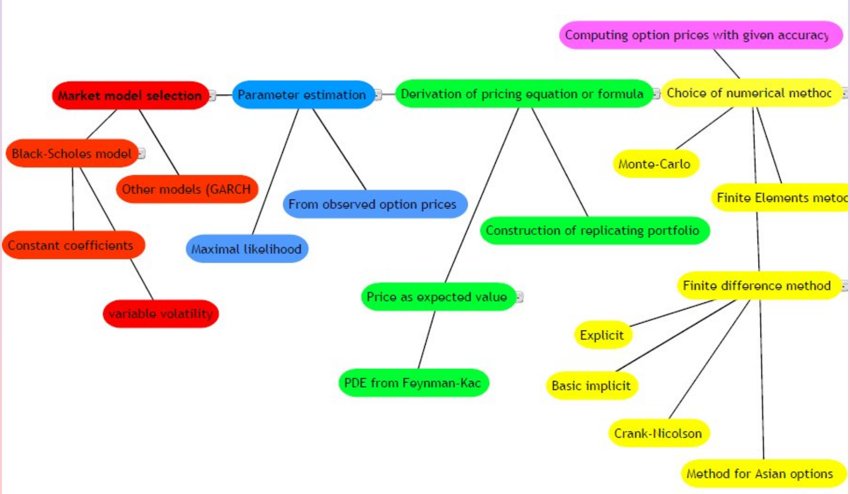
- Backtesting
- Main example: option pricing


Computing option prices with given accuracy
Market model selection
Parameter estimation
Derivation of pricing equation or formula
Choice of numerical methoc
Black-Scholes model
Other models (GARCH
From observed option prices
Monte-Carlo
Finite Elements metoc
Construction of replicating portfolio
Constant coefficients
Maximal likelihood
Finite difference method
Price as expected value
variable volatility
Explicit
PDE from Feynman-Kac
Basic implicit
Crank-Nicolson
Method for Asian options

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- Final grade: $\geq$ 90p -> A, 80-89.99p -> B, 70 - 79.99 points -> C and so on

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Example. An European option is the right to receive the amount $p(S(T))$ at time T.

Example. European call option is the right to buy one share of stock at time T for the price E, ie $p(s) = \max(s - E, 0)$.

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- In the case of some market models the option price is determined completely by the model and no arbitrage condition
- There are market models, for which the option prices are not determined completely but prices of different options have to be consistent with each other.

Black-Scholes market model

$$dS(t) = S(t)(\mu(t)\,dt + \sigma(S(t), t)\,dB(t))$$

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$B(t)$- the standard Brownian motion (Wiener process)
$B(t_2) - B(t_1)$ is Normally distributed with mean 0 and standard deviation $\sqrt{t_2 - t_1}$.

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- In most cases we have to solve approximately
- The simplest numerical method - Euler-Maruyama method
- Intuitively for non-intersecting time periods (t_{i-1}, t_i) we have

$$S(t_i) \approx S(t_{i-1}) + S(t_{i-1})(\mu(t_{i-1})h_i + \sigma(S(t_{i-1}), t_{i-1})X_i)$$
$$= S(t_{i-1})(1 + \mu(t_{i-1})h + \sigma(S(t_{i-1}), t_{i-1})X_i),$$

where $h_i = t_i - t_{i-1}$ and $X_j \sim N(0, \sqrt{h_i}),\ j = 1, 2, \ldots, N$ and X_i are independent normally distributed random variables. This relation enables us to simulate sample trajectories according to the market model.

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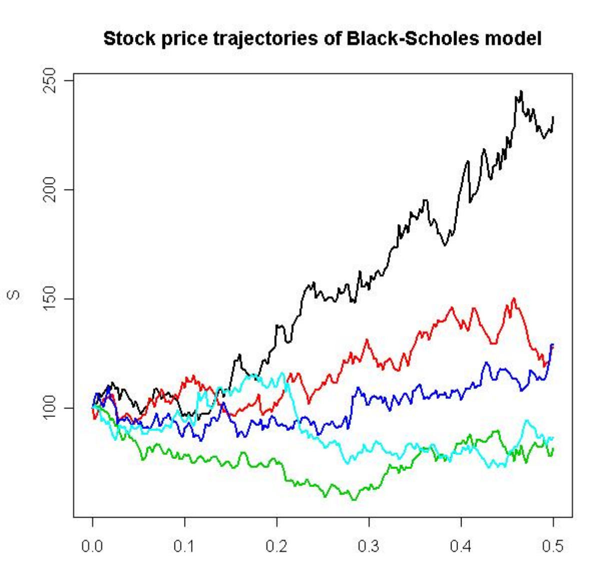
- The computations correspond to using Euler-Maruyama method for solving BS Stochastic Differential Equation (SDE)


Stock price trajectories of Black-Scholes model
S
100
150
200
250
0.0
0.1
0.2
0.3
0.4
0.5

Self-financing investment strategies

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No-arbitrage condition

It is not possible to make riskless profit on the market

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Corollary: If a self-financing portfolio produces exactly the same cash flows as holding an option, then the value of the portfolio and the option price have to be equal.

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- Let $f(x,t)$ be a function of two real numbers. What is

$$\frac{\partial}{\partial t} f(t^2, t)?$$

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Itō's formula

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$$df(Y(t),t) = \left(\frac{\partial f}{\partial t}(Y(t),t) + \frac{\beta(t)^2}{2}\frac{\partial^2 f}{\partial y^2}(Y(t),t)\right)\,dt + \frac{\partial f}{\partial y}(Y(t),t)\,dY(t).$$