Generalized Linear Models

Lecture 14. Summary

Generalized linear models

1. Framework and general theory

2. Models with continuous response

3. Models with binary response

4. Count data models

5. Tweedie models. Compound Poisson models

6. Models with censoring and truncation

Framework and general theory

- Form of a generalized linear model. Link and response function. Exponential family of distributions: likelihood function, mean and variance

- Estimation of the model parameters. Score function. Empirical and theoretical Fisher information matrix. Solving the maximum likelihood equations. Newton-Raphson and Fisher scoring methods

- Testing the hypothesis about the model. Likelihood ratio statistic, score statistic, Wald's statistic

- Goodness of fit of a model. Deviance

- Diagnostics of a GLM. Generalized residuals.

Models with continuous response

- Normally distributed response. Estimating the model. Diagnostics. Transformation of variables: Box-Cox family of transformations

- Gamma-distributed response (models with constant coefficient of variation). Estimating the model. Deviance. Link functions. Canonical link

- Models with log-normal and inverse Gaussian response

- Parametrization of distributions through the parameter of exponential family (θ)

Models with binary response

- Grouped and ungrouped data. Fitting the model. Goodness of fit (Hosmer-Lemeshow test, Pearson χ^2-test)

- Bernoulli and (scaled) binomial as members of exponential family

- Estimation of parameters. Separation problem and options to deal with it

- Theory of latent variable. Different choices for tolerance distribution. Different link functions. Canonical link. Interpretation of models

Count data models

- Poisson models. Apparent overdispersion. Quasi-likelihood. Grouped and ungrouped data. Interpretation of models

- Negative binomial models. Log-likelihood of the model. Link functions. Interpretation of models

- Models with zero-modifications. Derivation of the log-likelihood. Interpretation of models
  - Zero-inflated models (ZIP, ZINB)
  - Zero-altered / hurdle models (ZAP, ZANB)
  - Zero-truncated models (ZTP, ZTNB)

- Generalized Poisson models. ZIGP, ZTGP models

Compound Poisson models

- Compound Poisson models (CPo)

- Zero-inflated and zero-altered compound Poisson models (ZICPo, ZACPo)

- Poisson-Gamma compound distribution: Tweedie model Tw_p $(1 < p < 2)$

Models with censoring and truncation

- Censored and truncated count data models. Derivation of likelihood

- Survival models:
  - Survival/survivor function. Hazard function. Cumulative hazard
  - Distributions used. Parametric and semiparametric regression models (AFT and PH)
  - Log-likelihood with censored data
  - Proportional hazards model as Poisson model

- Censored/truncated continuous response:
  - Tobit model: idea, log-likelihood
  - Two-part models. Sample selection model