Topics of the lab: Fitting transfer function models.

In this lab we assume that we have two stationary processes Z_t and X_t and our aim is to predict next values of Z_t by using the information hidden in the process X_t as well as possible. For this, we have to determine, which lags of X_t should be included as regressors in the arima() command. More precisely, we are going to look for models of the form

$$Z_t = c_0 + \sum_{i=b}^{\infty} \beta_i X_{t-i} + \varepsilon_t, \tag{1}$$

where $b \geq 1$ and ε_t is an $ARMA$ process given by

$$\phi(B)\varepsilon_t = \theta(B)A_t,$$

where we additionally assume that the innovations A_t are independent of the values of the process X_t. Then also ε_t is independent of the process X_t. In order to estimate only a small number of parameters, we are going to assume that the function

$$\beta(x) = \sum_{i=b}^{\infty} \beta_i x^i$$

can be represented in the form

$$\beta(x) = x^b \frac{v(x)}{\delta(x)} = x^b \frac{\sum_{i=0}^{s} v_i x^i}{1 - \sum_{i=1}^{r} \delta_i x^i}.$$

Then we can also write the model (1) in the form

$$Z_t = \sum_{i=1}^{r} \delta_i Z_{t-i} + \sum_{i=0}^{s} v_i X_{t-b-i} + \eta_t,$$

where the process η_t is of the form

$$\phi(B)\eta_t = \delta(B)\theta(B)A_t.$$

The function $\beta(x)$ is called *transfer function* since it describes how the influence of the values of X_t is transfered to the process Z_t. The steps of fitting a transfer function model are as follows:

1. Estimate the values of β_i. For this, we fit an ARMA model to the X and use it for computing one-step ahead predictions of both X and Z (use the *same model* with the *same constants* that were estimated for X_t to predict values of Z_t) and find the cross correlations of prediction errors
2. Use the estimates of β_i to determine the lag b (which corresponds to the index of the first non-zero coefficient of β_i) and suitable values of r and s. The simplest case is if only a small number of coefficients β_i are different from 0 - then we can take $r = 0$ and $s = q - b + 1$, where q is the index of the last coefficient which is clearly different from 0. Other case which is easy to identify corresponds to the situations when the coefficients β_i start to decrease like being multiplied by a number with absolute value smaller than after certain lag q. In this case we take $r = 1$ and $s = q - 1$.
3. Fit an ARIMAX model to Z_t by using the lags $b, b+1, \ldots, b+s$ of X_t as regressors, use residuals to identify a model for ε_t.
4. Fit the complete model
5. Check the suitability fo the model
6. Compute forecast (up to b periods we can use the known values of the series X_t, for longer forecasts we have to forecast also the values of X_t).

Exercises of the lab:

1) Find a suitable transfer function model for the data in the file `lab13a.txt`
2) Find a suitable transfer function model for the data in the file `lab13b.dat`