# MTMM.00.179 Matemaatiline analüüs I Taylori valemi illustratsioonid

**Priit Lätt**

October 17, 2013

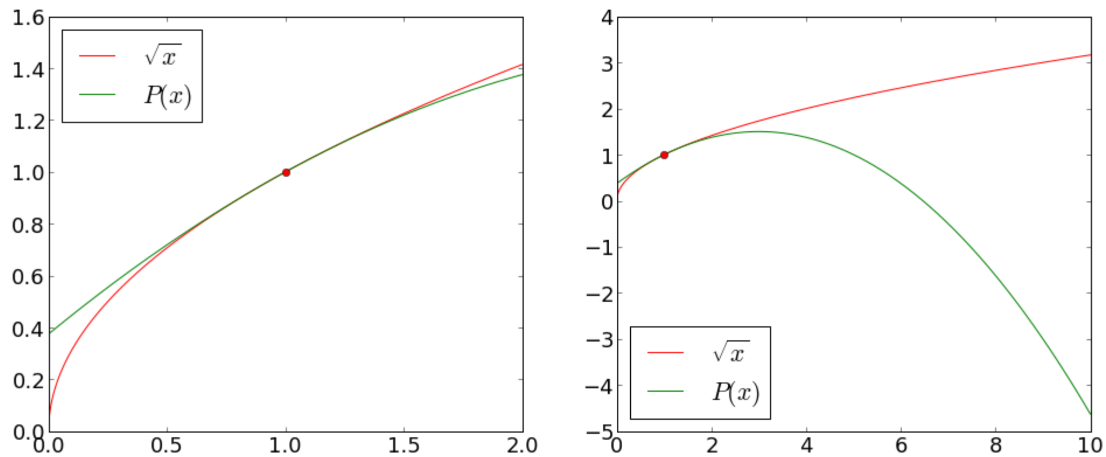
In [1]:
```
# Defineerime impordid, konstandid ja
# abimeetodid edaspidiseks kasutamiseks

import matplotlib.pyplot as plt
import numpy as np

pi = np.pi
sqrt = np.sqrt
e = np.e

def fig(x=16, y=14):
    plt.figure(figsize=(x, y))
    plt.rcParams['font.size'] = 18

def get_x(a, pm, step=0.001):
    return np.arange(a-pm, a+pm, step)
```

## Ülesanne 71. a)

Leidke funktsiooni $f(x) = \sqrt{x}$ Taylory $n$-järku polünoom punktis $a$, kus $a = 1,\ n = 2$.

In [2]:
```
a = 1

def yl71a(x):
    return sqrt(x)

def yl71a2(x):
    return 1 + (x - a)/2 - ((x - a)**2)/8

x1 = get_x(a, 1)
x2 = get_x(5, 5)
fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl71a(x1), 'r', label='$\sqrt{x}$')
plt.plot(x1, yl71a2(x1), 'g', label='$P(x)$')
plt.plot(a, yl71a(a), 'ro')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl71a(x2), 'r', label='$\sqrt{x}$')
plt.plot(x2, yl71a2(x2), 'g', label='$P(x)$')
plt.plot(a, yl71a(a), 'ro')
plt.legend(loc='best')
```

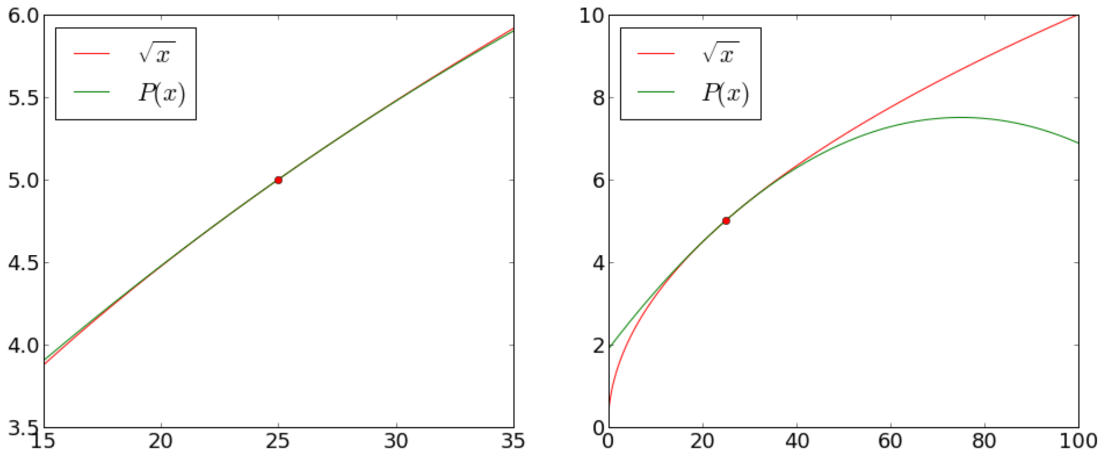
```
plt.show()
```




## Ülesanne 71. b)

Leidke funktsiooni $f(x) = \sqrt{x}$ Taylory $n$-järku polünoom punktis $a$, kus $a = 25,\ n = 2$.

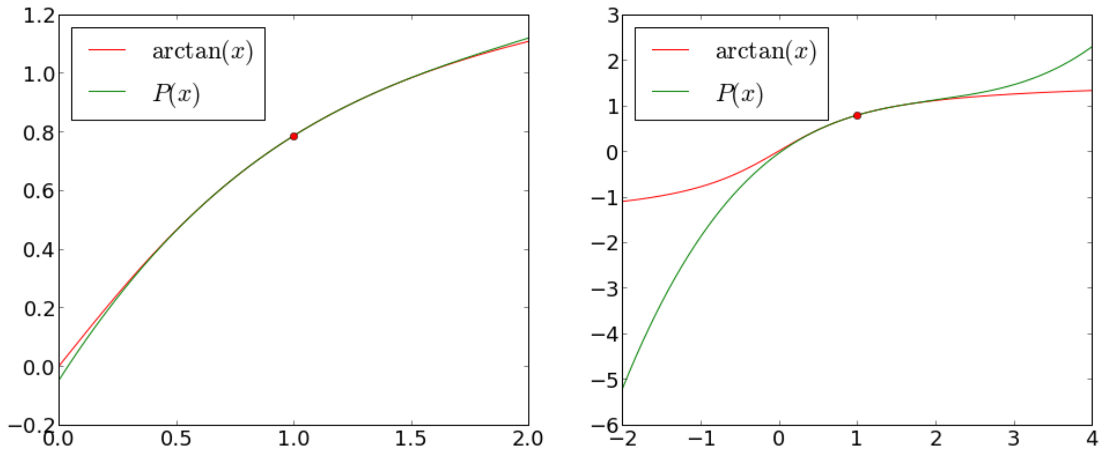
In [3]:
```
yl71b = yl71a
a = 25

def yl71b2(x):
    return 5 + (x - a)/10 - ((x - a)**2)/1000

x1 = get_x(a, 10)
x2 = get_x(a+25, 50)
fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl71b(x1), 'r', label='$\sqrt{x}$')
plt.plot(x1, yl71b2(x1), 'g', label='$P(x)$')
plt.plot(a, yl71b(a), 'ro')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl71b(x2), 'r', label='$\sqrt{x}$')
plt.plot(x2, yl71b2(x2), 'g', label='$P(x)$')
plt.plot(a, yl71b(a), 'ro')
plt.legend(loc='best')

plt.show()
```




## Ülesanne 71. c)

Leidke funktsiooni $f(x) = \arctan x$ Taylory $n$-järku polünoom punktis $a$, kus $a = 1,\ n = 3$.

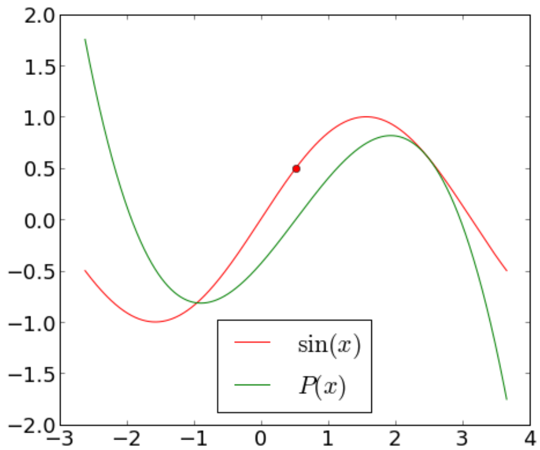
In [4]:
```
a = 1

def yl71c(x):
    return np.arctan(x)

def yl71c2(x):
    return pi/4 + (x - a)/2 - (x - a)**2/4 + (x - a)**3/12

x1 = get_x(a, 1)
x2 = get_x(a, 3)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl71c(x1), 'r', label=r'$\arctan{(x)}$')
plt.plot(x1, yl71c2(x1), 'g', label='$P(x)$')
plt.plot(a, yl71c(a), 'ro')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl71c(x2), 'r', label=r'$\arctan{(x)}$')
plt.plot(x2, yl71c2(x2), 'g', label='$P(x)$')
plt.plot(a, yl71c(a), 'ro')
plt.legend(loc='best')

plt.show()
```

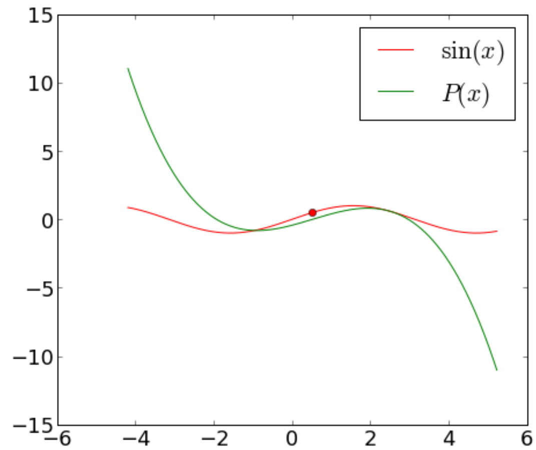



## Ülesanne 71. d)

Leidke funktsiooni $f(x) = \sin x$ Taylory $n$-järku polünoom punktis $a$, kus $a = \frac{\pi}{6}$, $n = 4$.

In [5]:
```
a = pi/6

def yl71d(x):
    return np.sin(x)

def yl71d2(x):
    d = lambda x: x - a
    return 1/2 + (sqrt(3)/2)*d(x)  \
        - (1/4)*d(x)**2 - (sqrt(3)/12)*d(x)**3 \
        + (1/48)*d(x)**4

x1 = get_x(a, pi)
x2 = get_x(a, 1.5*pi)
fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl71d(x1), 'r', label='$\sin{(x)}$')
plt.plot(x1, yl71d2(x1), 'g', label='$P(x)$')
plt.plot(a, yl71d(a), 'ro')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl71d(x2), 'r', label='$\sin{(x)}$')
plt.plot(x2, yl71d2(x2), 'g', label='$P(x)$')
plt.plot(a, yl71d(a), 'ro')
plt.legend(loc='best')

plt.legend(loc='best')

plt.show()
```

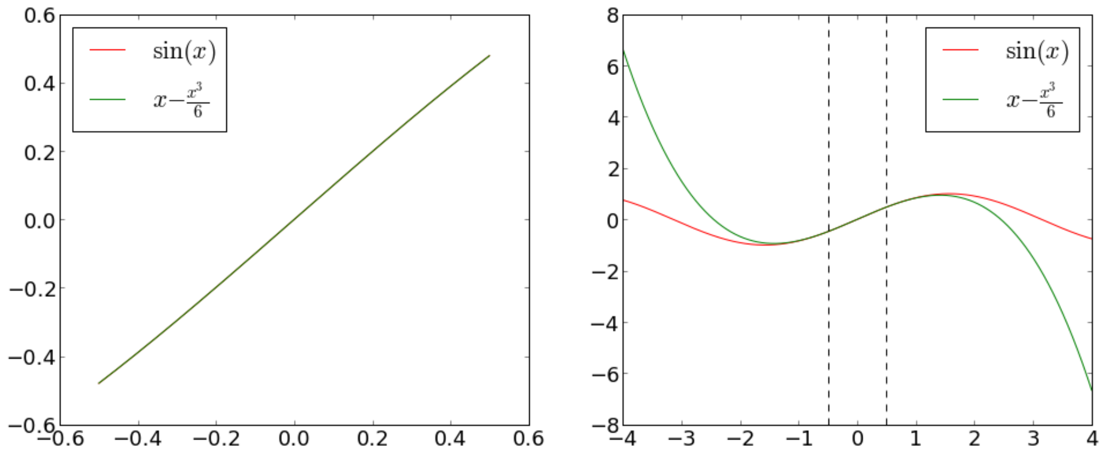



## Ülesanne 73. a)

Hinnake viga Taylori valemi kaudu saadud ligikaudses valemis: $\sin x \approx x - \frac{x^3}{6}$, kui $|x| \leq \frac{1}{2}$.

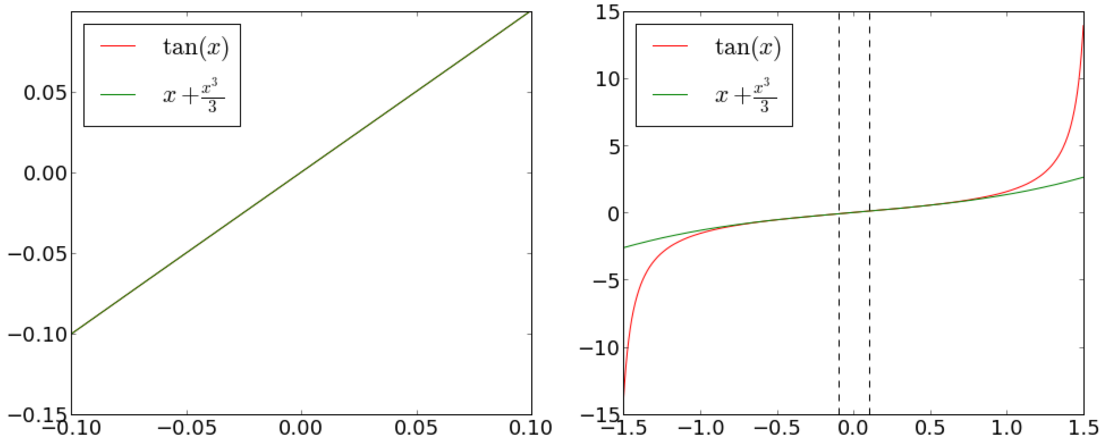
In [6]:
```
def yl73a(x):
    return np.sin(x)

def yl73a2(x):
    return x - x**3/6

x1 = get_x(0, 0.5)
x2 = get_x(0, 4)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl73a(x1), 'r', label='$\sin{(x)}$')
plt.plot(x1, yl73a2(x1), 'g', label=r'$x - \frac{x^3}{6}$')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl73a(x2), 'r', label='$\sin{(x)}$')
plt.plot(x2, yl73a2(x2), 'g', label=r'$x - \frac{x^3}{6}$')
plt.axvline(x=-0.5, color='k', ls='dashed')
plt.axvline(x=0.5, color='k', ls='dashed')
plt.legend(loc='best')

plt.show()
```




## Ülesanne 73. b)

Hinnake viga Taylori valemi kaudu saadud ligikaudses valemis: $\tan x \approx x + \frac{x^3}{3}$, kui $|x| \leq 0.1$.

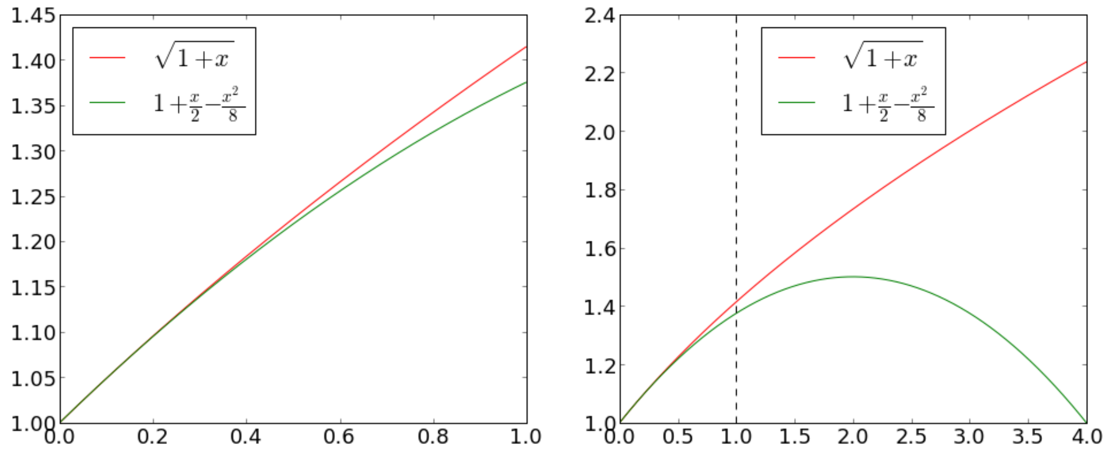
In [7]:
```
def yl73b(x):
    return np.tan(x)

def yl73b2(x):
    return x + x**3/3

x1 = get_x(0, 0.1, step=0.001)
x2 = get_x(0, 1.5, step=0.001)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl73b(x1), 'r', label=r'$\tan{(x)}$')
plt.plot(x1, yl73b2(x1), 'g', label=r'$x + \frac{x^3}{3}$')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl73b(x2), 'r', label=r'$\tan{(x)}$')
plt.plot(x2, yl73b2(x2), 'g', label=r'$x + \frac{x^3}{3}$')
plt.axvline(x=-0.1, color='k', ls='dashed')
plt.axvline(x=0.1, color='k', ls='dashed')
plt.legend(loc='best')

plt.show()
```




## Ülesanne 73. c)

Hinnake viga Taylori valemi kaudu saadud ligikaudses valemis: $\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8}$, kui $0 \leq x \leq 1$.

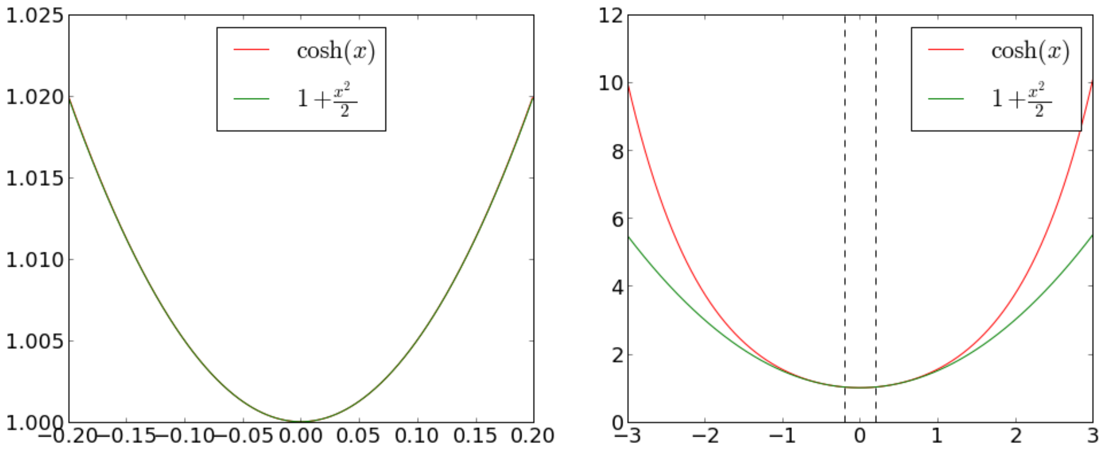
In [8]:
```
def yl73c(x):
    return sqrt(1+x)

def yl73c2(x):
    return 1 + x/2 - (x**2)/8

x1 = get_x(0.5, 0.5)
x2 = get_x(2, 2)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl73c(x1), 'r', label='$\sqrt{1+x}$')
plt.plot(x1, yl73c2(x1), 'g', label=r'$1+\frac{x}{2}-\frac{x^2}{8}$')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl73c(x2), 'r', label='$\sqrt{1+x}$')
plt.plot(x2, yl73c2(x2), 'g', label=r'$1+\frac{x}{2}-\frac{x^2}{8}$')
plt.axvline(x=0, color='k', ls='dashed')
plt.axvline(x=1, color='k', ls='dashed')
plt.legend(loc='best')

plt.show()
```




## Ülesanne 73. d)

Hinnake viga Taylori valemi kaudu saadud ligikaudses valemis: $\cosh x \approx 1 + \frac{x^2}{2}$, kui $|x| \leq 0.2$.

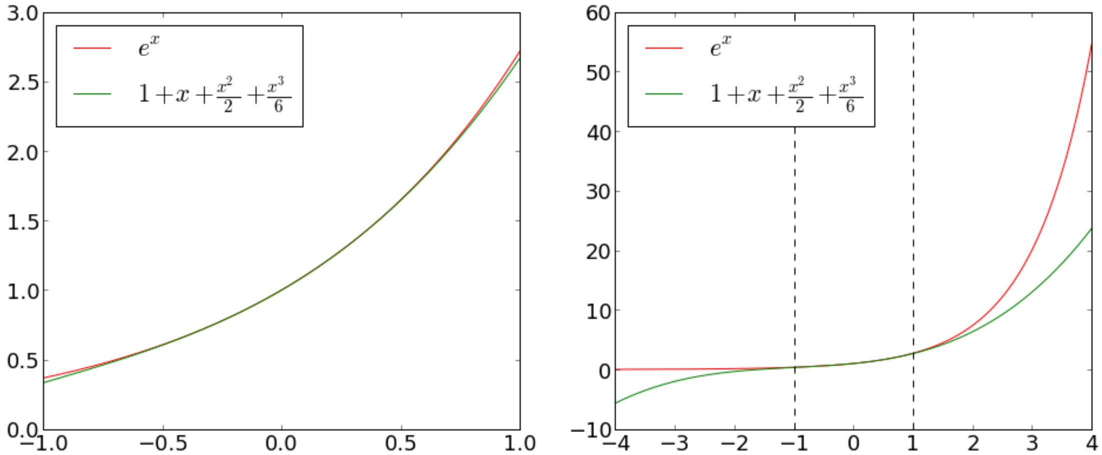
In [9]:
```
def yl73d(x):
    return np.cosh(x)

def yl73d2(x):
    return 1 + x**2/2

x1 = get_x(0, 0.2)
x2 = get_x(0, 3)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl73d(x1), 'r', label='$\cosh{(x)}$')
plt.plot(x1, yl73d2(x1), 'g', label=r'$1 + \frac{x^2}{2}$')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl73d(x2), 'r', label='$\cosh{(x)}$')
plt.plot(x2, yl73d2(x2), 'g', label=r'$1 + \frac{x^2}{2}$')
plt.axvline(x=-0.2, color='k', ls='dashed')
plt.axvline(x=0.2, color='k', ls='dashed')
plt.legend(loc='best')

plt.show()
```




## Ülesanne 73. e)

Hinnake viga Taylori valemi kaudu saadud ligikaudses valemis: $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, kui $|x| \leq 1$.

In [10]:
```
def yl73e(x):
    return e**x

def yl73e2(x):
    return 1 + x + x**2/2 +x**3/6

x1 = get_x(0, 1)
x2 = get_x(0, 4)

fig()

plt.figure(1)
plt.subplot(221)
plt.plot(x1, yl73e(x1), 'r', label='$e^x$')
plt.plot(x1, yl73e2(x1), 'g', label=r'$1+x+\frac{x^2}{2}+\frac{x^3}{6}$')
plt.legend(loc='best')

plt.subplot(222)
plt.plot(x2, yl73e(x2), 'r', label='$e^x$')
plt.plot(x2, yl73e2(x2), 'g', label=r'$1+x+\frac{x^2}{2}+\frac{x^3}{6}$')
plt.axvline(x=-1, color='k', ls='dashed')
plt.axvline(x=1, color='k', ls='dashed')
plt.legend(loc='best')

plt.show()
```


3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
$e^x$
$1+x+\frac{x^2}{2}+\frac{x^3}{6}$
60
50
40
30
20
10
0
−10
−4
−3
−2
−1
0
1
2
3
4
$e^x$
$1+x+\frac{x^2}{2}+\frac{x^3}{6}$