MTMM.00.005 Numerical Methods
Practical lesson 4: Secant method

March 5, 2019

To get a positive result, student has to solve all of the exercises given in practical lessons, explain the programming code and answer the questions.

The deadline for the second set of exercises is March 12th.

NB! In order to use the nonstandard packages as *numpy* and *matplotlib* one could use Command Prompt and type

```
H:\nm> "C:\Program Files\Anaconda3\python" <filename>.py
```

Here `H:\nm` is the folder for `<filename>.py`.

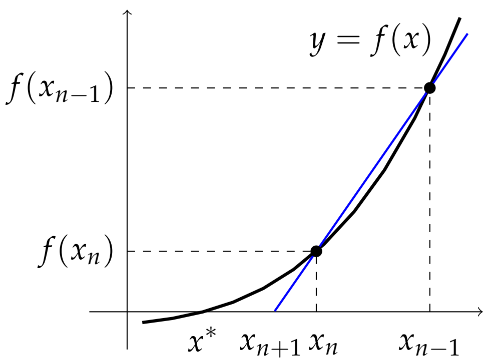
Secant method

Consider the equation $f(x) = 0$. Suppose that approximations x_{n-1} and x_n are already computed. To find x_{n+1} we replace the function $y = f(x)$ by the linear function

$$y = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}(x - x_n) + f(x_n)$$

joining $(x_{n-1}, f(x_{n-1}))$ and $(x_n, f(x_n))$. The approximation x_{n+1} is the x-intercept of this line, thus

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n), \quad n = 1, 2, \ldots .$$



Secant method can be viewed as a Newton method where the derivative $f'(x_n)$ is replaced by the divided difference

$$f(x_n, x_{n-1}) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

which is quite good approximation if x_{n-1} is close to x_n. However, secant method is not a special case of Newton's method nor fixed-point iteration method since two previous approximations x_n and x_{n-1} are used to compute x_{n+1}. At the beginning two initial values x_0 and x_1 are needed.

Exercise. Use secant method to find three smallest positive zeros of the function

$$f(x) = x \cos\left(\frac{x}{x - \alpha}\right).$$

Terminate iteration process if $|f(x_n)| \leq 10^{-8}$. Print out all computed values of x_n and the number of iterations needed to obtain the required accuracy.

Draw the graph of $f(x)$, $x \in [0, b]$, for some suitable b. Add the secants through the points $(x_0, f(x_0))$, $(x_1, f(x_1))$ such that their intercepts with x-axis (points x_2) are also marked out.