Simulation Methods in Financial Mathematics
Computer Lab 13
May 7, 2019

Goal of the lab: To practice using Quasi Monte Carlo methods for pricing financial options (using the Sobol sequences for computing approximate option prices).

1. Consider the problem of finding the price of the average strike option considered in lab 10 with total error that is less than 0.1 when a simple average is used for approximating the average stock price. Compute this price by using our usual procedure (knowing that the weak convergence rate is 1). Repeat the final computation with random numbers replaced by a suitable Sobol sequence. How much more accurate the final answer can be?

2. Recall the market model and option from Task 2, Lab 5:

   In reality, the future risk free interest rate is not a constant and can be considered to be a random variable. In the case Black-Scholes model with a random interest rate R the prices of European options can be computed as

   $$Price = E[\mathrm{e}^{-\int_0^T R(t)\,dt} p(S(T))],$$

   where

   $$dS(t) = S(t)((R(t) - D)\,dt + \sigma(t,s)\,dB_1(t))$$

   and $R(t)$ follows a suitable stochastic differential equation. We consider so called Cox-Ingersoll-Ross model

   $$dR(t) = a(b - R(t))\,dt + \sigma_2\sqrt{R(t)}\,dB_2(t),$$

   where a, b are constants, $B_1(t)$ and $B_2(t)$ are independent Brownian motions. So we have a system of stochastic differential equations for S and R. When computing the option prices we can replace the integral $\int_0^T R(t)$ with the product of the mean value or the interest rate and T. So, for computing the option price we should write a function that for a given values of parameters $D, \sigma, \sigma_2, T, a, b, m$ and n generates n pairs of the future stock prices $S(T)$ and mean values of the interest rates corresponding to the same trajectory. Compute the price of the call option described in the previous problem in the case of stochastic interest rate by using Euler's method with $m = 60$ time steps for solving the system of SDEs. Assume that $R(0) = 0.02,\ a = 0.5,\ b = 0.05,\ \sigma_2 = 0.2$.

   Repeat the computations of the lab by using a suitable Sobol sequence instead of random numbers.

3. **Homework 7** (Deadline 15.05.2019) Assume that two stock prices S_1 and S_2 follow the Black-Scholes stochastic differential equations

   $$dS_1(t) = S_1(t)(0.05\,dt + 0.5\,dB_1(t)),$$
   $$dS_2(t) = 0.05\,S_2(t)\,dt + \frac{S_2(t)}{0.01(S_2(t) - 30)^2 + 2}\,dB_2(t),$$

   where B_1 and B_2 are independent Brownian motions. Assume $S_1(0) = 100,\ S_2(0) = 31$. Consider an option that entitles the owner to receive at $T = 0.6$ the payment $\max(95 - S_1(T), S_2(T) - 30, 0)$. Knowing that the option price is given by

   $$P = E[e^{-0.05T}\max(95 - S_1(T), S_2(T) - 30, 0)],$$

find the option price with total error less than 0.1. Use the same number of time steps and the same number of generated values for computing an approximate option price by using QMC method with Sobol points. NB! The dimension of the Sobol points should be equal to the number of random numbers used for generating one value of the function under the expectation sign!