Simulation Methods in Financial Mathematics
Computer Lab 10

Goal of the lab:

- To learn to use Monte-Carlo method for pricing Asian options

An Asian option is an option which payoff depends on the average stock price. Let $A(T)$ be the average stock price for the period $[0, T]$ i.e.

$$A(T) = \frac{1}{T}\int_0^T S(t)\,dt.$$

The most typical payoff functions for Asian options are $p(s, a) = \max(s - a, 0)$ (i.e. at time T the value of the option is $p(S(T), A(T)) = \max(S(T) - A(T), 0)$; it is known as the average strike call option), $p(s, a) = \max(a - s, 0)$ (average strike put option), $p(s, a) = \max(a - E, 0)$ and $p(s, a) = \max(E - a, 0)$ (average price call and put options with strike price E). When the stock price is governed by the Black-Scholes market model, then the price of all the named options at time $t = 0$ can be calculated as the expected value

$$Price = E[e^{-rT}p(S(T), A(T))],$$

where $S(T)$ corresponds to the Black-Scholes market model with trend $\mu = r - D$. Thus to use the MC method we need to generate (in addition to the stock prices at time T) the average values of stock prices which depend also on the intermediate values of the stock price paths. The simplest way of calculating the average is using the average of $S(i\frac{T}{m}),\ i = 0, 1, \ldots, m - 1$; a better approximation can be calculated as

$$A(T) \approx \frac{1}{m}\sum_{i=1}^{m} S_{i-1}\left(1 + (r - D)\frac{T}{2m} + \frac{\sigma}{2}(B(t_i) - B(t_{i-1}))\right).$$

The idea leading to the improved formula is to write

$$\frac{1}{T}\int_0^T S(t)\,dt = \frac{1}{T}\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i} S(t)\,dt = \frac{1}{T}\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}(S(t_{i-1}) + \int_{t_{i-1}}^{t} dS(\tau))\,dt,$$

using the equation for $dS(\tau)$ and to approximate the resulting double integrals by replacing the integrands with their values at the beginning of the integration intervals.

We will assume that the Black-Scholes market model with constant parameters holds and fix $r = 0.1$, $D = 0$, $\sigma = 0.4$, $T = 0.5$, $S(0) = 100$. We will consider average strike calls and average price calls with strike price $E = 100$.

Tasks:

1. Write a generator which for a given value of n would generate n pairs of (terminal) stock price and average stock price. Calculate the stock prices using the exact formula (also use it when calculating the average stock prices); calculate the average as the mean of $S(i\frac{T}{m}),\ i = 0, 1, \ldots, m-1$. Find the weak convergence rate depending on m (by using the values $5, 10, 20, 40$ for m and a small enough MC error). For both options find m for which the error caused by the choice of m is less than 0.1 (using the result obtained for the weak convergence rate).

2. Repeat the task when the average price is calculated according to the improved formula. To study the rate of weak convergence use the values $2, 4, 6, 8$ for m and take 0.01 for the MC error.

3. **Homework 5. Important: you are expected to solve the homework on your own, without consulting with other students!** Use the optimal stratified sampling with 80 stratums to compute with the accuracy 0.001 (for $\alpha = 0.05$) the expected value of

$$U = \frac{\sin^2(2X - 5Y + Z)}{1 + 0.02X^2}$$

where X, $Y$and Z are independent random variables from the normal distribution $N(0, \sigma = 3)$. The stratification should be based on the values of the variable $V = 2X - 5Y + Z$. **In the solution script, write as a comment the following declaration:**
I certify that I have solved this homework problem myself without help from other students or persons except the lecturer of the course and that I have not copied in part or whole or otherwise plagiarised the work of other students.

Your Name, date.