Topics of the lab: Partial ARIMA models and multiplicative seasonal ARIMA models

Often it is not possible to find a suitable ARIMA(p,d,q) model with reasonably small number of parameters, but there is a reason to believe that the behaviour of a given series is a result of a relatively small number of effects appearing on different time scale (for example, monthly and yearly effects or hourly and weekly effects). This means that we want to find a model with a small number of non-zero parameters, but some of them may correspond to large time lags. In such cases it is reasonable to fit partial ARIMA models (where some parameters are forced to be 0) or seasonal multiplicative ARIMA models.

Multiplicative seasonal ARIMA models with period s are models of the form

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D Z_t = \theta(B)\Theta(B^s)A_t,$$

where

$$\phi(x) = 1 - \sum_{i=1}^{p} \phi_i x^i,\ \Phi(x) = 1 - \sum_{i=1}^{P} \Phi_i x^i,$$
$$\theta(x) = 1 - \sum_{i=1}^{q} \theta_i x^i,\ \Theta(x) = 1 - \sum_{i=1}^{Q} \Theta_i x^i.$$

Intuitively it is easy to understand seasonal multiplicative models of the form ARIMA(p,d,q)x(0,1,0)$_s$, which correspond to usual ARIMA modelling of seasonal differences. Multiplicative seasonal ARIMA models are special cases of partial ARIMA models but have smaller number of parameters.

Exercises:

1. Find suitable partial or seasonal ARIMA models for time series in the file `lab10.txt` (available from Moodle). To fit a partial ARIMA model, it is necessary to specify a parameter `fixed` indicating values which should be set to 0. This parameters should be a vector of length $p+q+1$ if $d=0$ and the parameter `include.mean` is TRUE (which is the default value), and $p+q$ otherwise. The first p components denote AR coefficients, next q components correspond to MA coefficients and the last one (if present) denotes the mean value μ. Only the parameters denoted by NA are fitted, other parameters are set to given values (which should be 0 in the case of a partial ARIMA model). For example, to fit the model

$$Z_t - \mu = \phi_4(Z_{t-4} - \mu) + A_t - \theta_1 A_{t-1} - \theta_3 A_{t-3}$$

one should use the command
`arima(Z,order=c(4,0,3),fixed=c(0,0,0,NA,NA,0,NA,NA))`.
For fitting a multiplicative seasonal model one should specify an additional parameter
`seasonal=list(order=(P,D,Q),period=s)`,
to the arima command (here s is the number of observations in the period).