Teist järku joonte ja pindade parametriseeringud

Teist järku jooned

Ellips $\varepsilon = \left\{(x,y)\colon \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1\right\}.$

Poolteljed $a, b > 0$, kusjuures $a \geqslant b$. Tähistame $p = \dfrac{b^2}{a}$, $c = \sqrt{a^2 - b^2}$, $e = \dfrac{c}{a}$.

$$\begin{aligned}
\varepsilon &= \left\{\left(x, b\sqrt{1-\frac{x^2}{a^2}}\right) : x \in [-a,a]\right\} \cup \left\{\left(x, -b\sqrt{1-\frac{x^2}{a^2}}\right) : x \in (-a,a)\right\} = \\
&= \{(a\cos t, b\sin t)\colon t \in [0, 2\pi)\} = \\
&= \left\{\left(\frac{p}{1-e\cos t}\cdot\cos t - c, \frac{p}{1-e\cos t}\cdot \sin t\right) : t \in [0,2\pi)\right\}.
\end{aligned}$$

Hüperbool $\gamma = \left\{(x,y)\colon \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1\right\}.$

Poolteljed $a, b > 0$. Tähistame $p = \dfrac{b^2}{a}$, $c = \sqrt{a^2 + b^2}$, $e = \dfrac{c}{a}$.

$$\begin{aligned}
\gamma &= \left\{\left(a\sqrt{1+\frac{y^2}{b^2}}, y\right) : y \in \mathbb{R}\right\} \cup \left\{\left(-a\sqrt{1+\frac{y^2}{b^2}}, y\right) : y \in \mathbb{R}\right\} = \\
&= \left\{\left(x, b\sqrt{\frac{x^2}{a^2}-1}\right) : |x| \geqslant a\right\} \cup \left\{\left(x, -b\sqrt{\frac{x^2}{a^2}-1}, y\right) : |x| \geqslant a\right\} = \\
&= \{(a\,\mathrm{ch}\,t, b\,\mathrm{sh}\,t)\colon t \in \mathbb{R}\} \cup \{(-a\,\mathrm{ch}\,t, b\,\mathrm{sh}\,t)\colon t \in \mathbb{R}\} = \\
&= \left\{\left(\frac{p}{1-e\cos t}\cdot\cos t + c, \frac{p}{1-e\cos t}\cdot\sin t\right) : t \in [0,2\pi) \setminus \left\{\arccos\frac{1}{e}, 2\pi - \arccos\frac{1}{e}\right\}\right\}.
\end{aligned}$$

Parabool $\pi = \{(x,y)\colon y^2 = 2px\}.$

Parameeter $p > 0$.

$$\begin{aligned}
\pi &= \left\{\left(\frac{y^2}{2p}, y\right) : y \in \mathbb{R}\right\} = \\
&= \left\{\left(x, \sqrt{2px}\right) : x \geqslant 0\right\} \cup \left\{\left(x, -\sqrt{2px}\right) : x > 0\right\} = \\
&= \left\{\left(\frac{p}{1-\cos t}\cdot\cos t - \frac{p}{2}, \frac{p}{1-\cos t}\cdot\sin t\right) : t \in (0,2\pi)\right\}.
\end{aligned}$$

Teist järku pinnad

Ellipsoid $\varepsilon = \left\{(x, y, z)\colon \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1\right\}.$

Poolteljed $a, b, c > 0$.

$$\begin{aligned}\varepsilon &= \left\{\left(x, y, c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\right) : (x, y) \in [-a, a] \times [-b, b]\right\} \cup \\ &\qquad \cup \left\{\left(x, y, -c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\right) : (x, y) \in [-a, a] \times [-b, b]\right\} = \\ &= \{(a \cos u \sin v, b \sin u \sin v, c \cos v)\colon (u, v) \in [0, 2\pi) \times [0, \pi]\}\,.\end{aligned}$$

Koonus $\varkappa = \left\{(x, y, z)\colon \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 0\right\}.$

Konstandid $a, b, c > 0$.

$$\begin{aligned}\varkappa &= \left\{\left(x, y, c\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}\right) : (x, y) \in \mathbb{R}^2\right\} \cup \left\{\left(x, y, -c\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}\right) : (x, y) \in \mathbb{R}^2\right\} = \\ &= \{(at \cos u, bt \sin u, ct) : (t, u) \in \mathbb{R} \times [0, 2\pi)\}\,.\end{aligned}$$

Ühekattene hüperboloid $\gamma = \left\{(x, y, z)\colon \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1\right\}.$

Konstandid $a, b, c > 0$.

$$\begin{aligned}\gamma &= \left\{\left(a\sqrt{1 + \frac{z^2}{c^2}} \cos u, b\sqrt{1 + \frac{z^2}{c^2}} \sin u, z\right) : (z, u) \in \mathbb{R} \times [0, 2\pi)\right\} = \\ &= \{(a \cos u \operatorname{ch} v, b \sin u \operatorname{ch} v, c \operatorname{sh} v) : (u, v) \in [0, 2\pi) \times \mathbb{R}\} = \\ &= \{(a \cos u - at \sin u, b \sin u + bt \cos u, ct) : (t, u) \in \mathbb{R} \times [0, 2\pi)\} = \\ &= \{(a \cos u + at \sin u, b \sin u - bt \cos u, ct) : (t, u) \in \mathbb{R} \times [0, 2\pi)\}\,.\end{aligned}$$

Kahekattene hüperboloid $\gamma = \left\{(x, y, z)\colon \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = -1\right\}.$

Konstandid $a, b, c > 0$.

$$\begin{aligned}\gamma &= \left\{\left(a\sqrt{\frac{z^2}{c^2} - 1} \cos u, b\sqrt{\frac{z^2}{c^2} - 1} \sin u, z\right) : |z| \geqslant c, u \in [0, 2\pi)\right\} = \\ &= \{(a \cos u \operatorname{sh} v, b \sin u \operatorname{sh} v, c \operatorname{ch} v) : (u, v) \in [0, 2\pi) \times \mathbb{R}\} \cup \\ &\qquad \cup \{(a \cos u \operatorname{sh} v, b \sin u \operatorname{sh} v, -c \operatorname{ch} v) : (u, v) \in [0, 2\pi) \times \mathbb{R}\}\,.\end{aligned}$$

Elliptiline paraboloid $\pi = \left\{(x, y, z)\colon \frac{x^2}{p} + \frac{y^2}{q} = 2z\right\}$.

Konstandid $p, q > 0$.

$$\pi = \left\{ \left(\sqrt{2pz} \cos u,\, \sqrt{2qz} \sin u,\, z \right) \,:\, (z, u) \in [0, \infty) \times [0, 2\pi) \right\}.$$

Hüperboolne paraboloid $\pi = \left\{(x, y, z)\colon \frac{x^2}{p} - \frac{y^2}{q} = 2z\right\}$.

Konstandid $p, q > 0$.

$$\begin{aligned}
\pi &= \left\{ \left(\sqrt{2pz}\, \mathrm{ch}\, u,\, \sqrt{2qz}\, \mathrm{sh}\, u,\, z \right) \,:\, (z, u) \in [0, \infty) \times \mathbb{R} \right\} \cup \\
&\quad \cup \left\{ \left(-\sqrt{2pz}\, \mathrm{ch}\, u,\, \sqrt{2qz}\, \mathrm{sh}\, u,\, z \right) \,:\, (z, u) \in [0, \infty) \times \mathbb{R} \right\} \cup \\
&\quad \cup \left\{ \left(\sqrt{-2pz}\, \mathrm{sh}\, u,\, \sqrt{-2qz}\, \mathrm{ch}\, u,\, z \right) \,:\, (z, u) \in (-\infty, 0] \times \mathbb{R} \right\} \cup \\
&\quad \cup \left\{ \left(\sqrt{-2pz}\, \mathrm{sh}\, u,\, -\sqrt{-2qz}\, \mathrm{ch}\, u,\, z \right) \,:\, (z, u) \in (-\infty, 0] \times \mathbb{R} \right\} = \\
&= \left\{ ((u+v)\sqrt{p},\, (u-v)\sqrt{q},\, 2uv) \,:\, (u, v) \in \mathbb{R} \times \mathbb{R} \right\}.
\end{aligned}$$