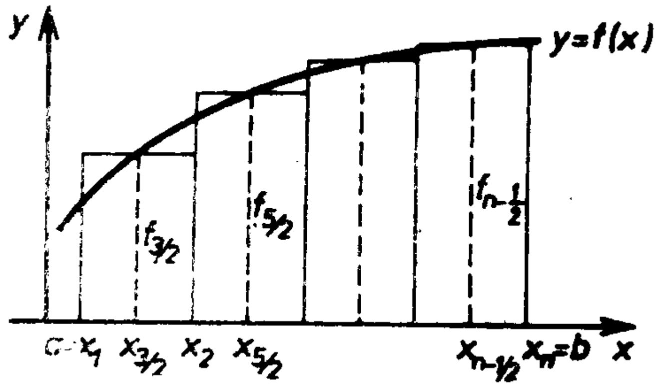

Joonis 12

Nii jõuame ristkülikvalemini

$$\int_a^b f(x)\,dx = h\sum_{k=1}^{n-1} f_{k+1/2} + \frac{h^3}{24}\sum_{k=1}^{n-1} f''(\xi_k),$$

mida illustreerib joonis 12. Funktsiooni f'' pidevuse tõttu

$$\frac{1}{n-1}\sum_{k=1}^{n-1} f''(\xi_k) = f''(\xi),$$

kus $\xi \in [a, b]$. Seega saame ristkülikvalemi kirjutada kujul

$$\int_a^b f(x)\,dx = h(f_{3/2}+f_{5/2}+\ldots+f_{n-1/2}) + \frac{b-a}{24}h^2 f''(\xi), \tag{11}$$

sest $(n-1)h = b-a$.

Kui $f \in C^4[a, b]$, siis saame järgmisel teel leida ristkülikvalemi jääkliikme peaosa. Valemite (9) ja (10) põhjal

$$\int_a^b f(x)\,dx = h\sum_{k=1}^{n-1} f_{k+1/2} + \frac{h^3}{24}\sum_{k=1}^{n-1} f''(x_{k+1/2}) + \frac{b-a}{1920}h^4 f^{IV}(\xi_1),$$

kus $\xi_1 \in [a, b]$. Teise summa avaldame valemi (11) abil

$$h\sum_{k=1}^{n-1} f''(x_{k+1/2}) = \int_a^b f''(x)\,dx - \frac{b-a}{24}h^2 f^{IV}(\xi_2) =$$

$$= f'(b) - f'(a) - \frac{b-a}{24}h^2 f^{IV}(\xi_2), \tag{12}$$

kus $\xi_2 \in [a, b]$. Seega

$$\int_a^b f(x)\,dx = h\sum_{k=1}^{n-1} f_{k+1/2} + \frac{h^2}{24}[f'(b) - f'(a)] + O(h^4).$$