Computational Finance, Spring 2023
Computer Lab 14

The aim of the lab is to check how well European options can be replicated by trading.

If the Black-Scholes market model holds, then by arguments used in the lecture we know that every European and American option can be replicated by a self-financing trading strategy that uses the sum of money equal to the option price as starting capital and holds at each time moment $\frac{\partial v}{\partial s}(S(t), t)$ shares of the stock. We assume that short selling (holding negative number of stocks) is possible and that stocks are infinitely divisible, so that any fraction of a stock can be held in a portfolio.

We shall simulate the trading by using the historical data of Cisco stock prices (dividend adjusted daily closing prices) for one year.

1. Read the adjusted closing price data for Cisco stock for the period 10 May 2022-9 May 2023 (both ends included) into python. We'll assume that the Black-Scholes market model holds with constant volatility. Estimate the volatility by using the first half of the data.
2. Consider an European put option with exercise price equal to the last stock price used for parameter estimation and exercise time half a year. Compute the price of the option (using $r = 0.02$) and it's derivative with respect to the stock price. For computing the derivative, find from the internet a formula for it (hint: derivative of a stock price with respect to stock price is called delta in finance literature).
3. Perform trade simulation for self financing portfolio which should theoretically replicate the option and check the outcome. For this:
    - Let us set up a self-financing portfolio. It consists of an bank account and a stock holding; the initial stock holding is equal to the derivative found in the previous step and the bank account is initially option price minus the money under the stock (ie stock holding times the stock price).
    - For each day until expiry of the option we now simulate the change in the portfolio: we'll find the number of stocks we should have on that day (by finding the derivative of the option price at the current time and stock price) and modify the bank account by the interest earned and the money coming from the change of the stock holdings.
    - If the theory is good enough the difference between the portfolio value and the payoff function should be practically 0 at the final time.
4. Modify the procedure for American put options. Then a numerical method should be used for pricing the option and finding it's derivative. Replication works reasonably well if the value of our portfolio at each time moment never becomes much less than the pay-off value at that time moment and when there exists a time when the value of the portfolio and the value of the pay-off function of the stock price at that time moment are practically equal.