Computational Finance
Computer Lab 1

The aim of the Lab is to get acquainted with the Python programming language and some commands of `numpy` library. Try to solve the problems by consulting the document "Python commands needed for this course" if necessary. If you run into trouble, please ask for help immediately!
Exercises:

1. Suppose the number of foxes and the number of rabbits on an island at different years behave according to equations

$$f_{i+1} = \text{floor}(0.8\, f_i + \frac{f_i\, r_i}{1000}),$$
$$r_{i+1} = \text{floor}(1.5 r_i - \frac{f_i r_i}{50}),$$

   where floor(x) is the largest integer which is not larger than x (corresponding `numpy` function is `floor()`). Suppose the current number of rabbits is 150 and the current number of foxes is 20. Compute the number of foxes and the number of rabbits for the next 30 years and store the results in `numpy` arrays `f` and `r`.

2. Produce a plot of points $(f_i, r_i),\ i = 0, 1, \ldots, 30$.

3. Generate $n = 20$ values from the standard normal distribution and store the values in a vector `x`. Compute the values of the vector `y` defined by

$$y_0 = x_0;\ y_{n-1} = x_{n-1};\ y_i = \frac{x_{i-1} + x_i + x_{i+1}}{3},\ i = 1, 2, \ldots, n-2$$

   without using **`for`** or **`while`** cycles.

4. Define a function $g(x) = \sin(2x) + \frac{\cos(x)}{2x}$. Plot the graph of the function for $1 \le x \le 10$ by using the values of the function at 101 points.

5. Write a function `simpleMax(f,a,b,n)` that computes approximately the maximal value of the function f over the interval $[a, b]$ by evaluation it at the points $a,\ a + h,\ a + 2h, \ldots, b$, where $h = \frac{b-a}{n}$. Use the function `simpleMax` to compute approximately the maximal value of the function g from the previous exercise for $x \in [1, 3]$ in the case $n = 200$ (answer should be 1.1794485797597516).

6. Write a function `Simple2dMax(f,a,b,m,c,d,n)` which computes approximately the maximum of a function of 2 variables by computing the maximal value at the points

$$x_i = a + ih_1,\ y_j = c + jh_2,\ i = 0, \ldots, m,\ j = 0, \ldots, n,$$

   where $h_1 = \frac{b-a}{m},\ h_2 = \frac{d-c}{n}$. Test correctness of the function by computing the maximal value of $u(x, y) = x^2 + xy - 2y^2$ over the unit square $0 \le x \le 1,\ 0 \le y \le 1$ in the case $m = 10,\ n = 20$ (the answer should be 1.125).

7. Extra problem. Use the function `np.meshgrid()` for implementing the function `Simple2dMax()` without any **`for`** cycles.