Computational Finance
Computer Lab 5

The aim of the Lab is to learn to determine market parameters from the prices of traded options. If we make assumptions about the market behavior or about the methods of option pricing, we get functions that for a given set of market (or option pricing) parameters give theoretical values of every concrete option. Black-Scholes formulas are examples of such functions that for given value of the volatility σ and for given put or call option parameters (exercise price, duration) give the price of the option. Whenever we have such functions (which can be explicit formulas or some computer programs that compute the prices) and there are available prices of some traded options we can try to determine the unknown parameters from the known option prices. More precisely, suppose that we know the current prices $V_1, V_2, \ldots, V_m$ of m different options and that $f_i(\theta)$ are the functions that give the option prices for the (unknown) market parameters θ. Then we have m equations:

$$f_i(\theta) = V_i,\ i = 1, \ldots, m.$$

Usually the number of unknown market parameters is much smaller than the number of available option prices, so the system of equations may be solved in the least squares sense by minimizing the function

$$F(\theta) = \frac{1}{2}\sum_{i=1}^{m}(f_i(\theta) - V_i)^2.$$

Let us use the information about ask prices of 4.4-months call options (expiration on July 17, 2020) for AstraZeneca Plc stock available from Moodle (from `http://finance.yahoo.com` on March 9, 2020).

Exercise 1. (Implied volatility) Let us assume that the Black-Scholes market model with constant volatility holds, then call option prices can be computed by Black-Scholes formula. Assume $r = 0.01$ and $D = 0$, then the only unknown parameter is σ. Since we have only one unknown parameter, only one equation is needed to determine the value of σ and if the assumption about the market model is correct, then every known option price should give the same value of σ. Use the equation solver `fsolve` from `scipy.optimize` to find a value of σ for each of 10 actual option prices (computed as the average of the Bid and Ask prices) corresponding to 10 strike prices closest to the current share price (as of March 6, 2020). Show the dependence of found σ values on the exercise price on a graph. In order to do this, for each value of the exercise price define a function of one argument σ that computes the difference of the corresponding theoretical call option price and the observed price of the option and use this function as an input for the command `optimize.fsolve` together with a suitable initial guess for the parameter σ.

Exercise 2. Define a function that for a given value of σ computes the sum of squares of differences of the theoretical and observed option prices and use a minimizer from `scipy.optimize` to find the least squares estimate of σ. For this σ, find the largest difference between the theoretical and observed option prices.

Exercise 3. Consider Black-Scholes market model with the non-constant volatility

$$\sigma(s,t) = |\theta_0 + \theta_1 \frac{1}{1 + 0.1(s-44)^2}|.$$

From the course web page you can download a module `lab5solver` that contains a function `lab5solver(theta,E,S0,T,r)` that for a given values of E, r, T and $S0$ and for a given parameter vector θ computes the theoretical price of a call option. Use the function and the option price data to find suitable values of θ_0 and θ_1.