Home reading and exercises: Exchangeability and de Finetti's theorem

Read the following:

- LN, p. 7-10
- Farago, *De Finetti Style Theorems With Applications to Network Analysis* (2021), p. 1-10

and answer the questions below.

The following sources are optional, but you may find them interesting:

- BDA, ch. 5.2. (p. 105-108)
- Diaconis, *Finite Forms of de Finetti's Theorem* (1977)
- Diaconis & Freedman, *De Finetti's Theorem for Markov Chains* (1980)
- Zaman, *Urn Models for Markov Exchangeability* (1984)
- Zaman, *A Finite Form of De Finetti's Theorem for Stationary Markov Exchangeability* (1986)
- Diaconis et al., *De Finetti Priors Using Markov Chain Monte Carlo Computations* (2015)
- Fortini & Petrone, *Predictive Characterization of Mixtures of Markov Chains* (2015)
- Berta et al., *A Third Information-Theoretic Approach to Finite de Finetti Theorems* (2024)
- Wikipedia, Exchangeable random variables, Exchangeable σ-algebra, de Finetti's theorem

Exchangeability

1. What is the formal definition of exchangeability?

2. How does exchangeability differ from independent and identically distributed random variables (i.i.d.)?
3. Provide an example of an infinite exchangeable sequence that is not i.i.d.
4. Assume that we have a finite population of size N and we randomly draw
   - a sample of size n with replacement,
   - a sample of size n without replacement.
   Is the sample i.i.d.? Is it exchangeable?

de Finetti theorems

5. State de Finetti's theorem for infinite binary sequences and deFinetti-Hewitt-Savage theorem for more general sequences.
6. Try to think or find philosophical or practical implications of de Finetti's theorem.
7. How does de Finetti's theorem change for finite exchangeable sequences? Mention some of the finite de Finetti theorems.

8. Mention some of the generalizations of exchangeability and de Finetti's theorem to more complex structures.

Exercises

1. Exercise 2 in LN, p. 11